TRIGONOMETRICAL LEVELLING

Unit 2

2.1 INTRODUCTION

Trigonometrical levelling is an indirect method of levelling. The relative elevations of various points are determined from observed vertical angles and horizontal distances by the use of certain trigonometrical relations. The vertical angles are usually measured with a theodolite and the horizontal distances are measured with a tape or chain or calculated indirectly. This method is also known as Heights and distances.

Trigonometrical levelling can be divided into two groups.

1. Plane trigonometrical levelling.
2. Geodetic trigonometrical levelling.

In plane trigonometrical levelling, the principles of plane surveying are used. It is assumed that the distances between points are not large and the effect of curvature and refraction is applied linearly to the calculated heights and distances. If the distances are small, the effect of curvature and refraction is neglected.

In geodetic trigonometrical levelling, the principle of plane surveying cannot be used as the distances between the points are very large. The correction due to curvature and refraction are applied directly to the observed angles.

Using plane trigonometrical levelling, elevations of points like top of chimney, church spire, top of temple etc., are determined.

The following situations under which trigonometrical surveying may be used:

1. When the base of the object is accessible, but it is difficult to measure the height of the object directly with a tape. Example: Tower.
2. When the base of the object is inaccessible due to obstacle between instrument and object. Example: River.

3. When the base of the object is inaccessible due to undulating ground or object is located on the top of hill.

2.2 FINDING ELEVATION OF OBJECTS

To find the elevation of objects, the following two cases are considered:

Case: 1 Base of the object is accessible.

Case: 2 Base of the object is inaccessible:

- Single plane method.
- Double plane method.

2.3 ELEVATION OF AN OBJECT WHEN IT'S BASE IS ACCESSIBLE

In this case the base of the object say chimney is accessible. The horizontal distance (D) between the object (Q) and the instrument station (B) can be measured directly using a tape. This method is usually employed when the distance D is small.

Let, Q be the top of the object whose elevation is required. The following field procedure is used:

1. Set up the theodolite over B and level it accurately with reference to altitude bubble.
2. Take a staff reading (S) on B.M with the line of sight horizontal to determine the elevation of line of sight.
3. Direct the telescope towards Q and observe the vertical angle (α) Face left and face right observations should be taken to eliminate instrumental errors.

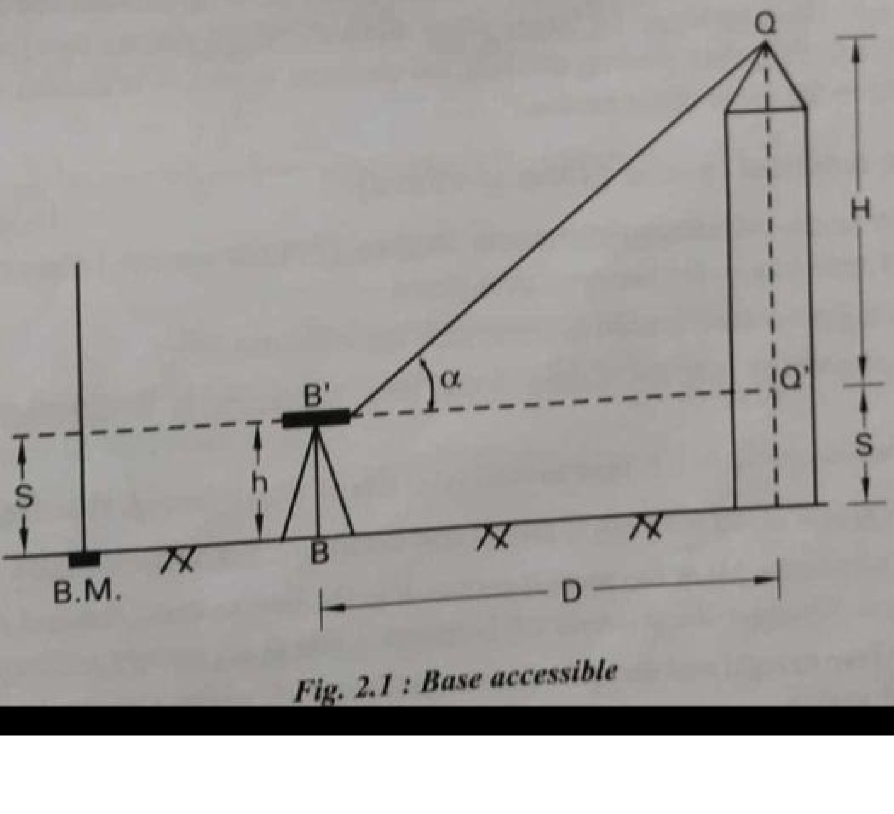

Fig. 2.1 : Base accessible

Let, h = height of the instrument at B

H = height of object (Q) above horizontal line of sight = $B'Q'$

In the triangle $B'Q'Q$,

$$H = D\ tan\alpha$$

Therefore,

R.L. of Q = R.L. of instrument axis + H

R.L. of instrument axis = R.L. of $B + h$ (or) R.L. of B.M. + S

If the distance D is large, combined correction for curvature and refraction should be applied to the calculated height.

The combined correction for curvature and refraction, $C = 0.06735D^2$

Where, D is the horizontal distance in kilometers. Its sign is (+)ve for angle of elevation and (–) ve for angle of depression.

Then, R.L. of Q = R.L. of instrument axis + $H + C$

2.4 ELEVATION OF AN OBJECT WHEN IT'S BASE IS INACCESSIBLE

If the base of a high object such as chimney is not accessible, the horizontal distance between the base of the object and the instrument station cannot be measured. In this case, two instrument stations are chosen on a fairly level ground.

The distance between the two instrument stations is measured by a tape and the vertical angles are observed from the two stations to determine the elevation of top of the object. If the two instrument stations so chosen lie in the same vertical plane passing through the elevated object, it is known as "single plane method" or "same plane method". If the chosen two instrument stations do not lie in the vertical plane passing through the elevated object, it is known as "Double plane method" or "Not in the same plane method".

Single plane method (same plane method)

Let A and B be the two chosen instrument stations. Q be the elevated object whose elevation is required. A, B and Q lie in the same vertical plane.

The following procedure is used to determine the elevation of Q.

1. Set up the theodolite over the station A and level it accurately with respect to the altitude bubble.
2. Direct the telescope with left face towards the top of the object Q. Bisect Q accurately and clamp both plates. Read verniers C and D and determine the vertical angle α_1.
3. Plunge the telescope. Mark the second station B in the line so that Q, A and B are in the same vertical plane. Measure the distance (d) between A and B accurately with a tape.
4. Change the face to right and measure the vertical angle α_1 again. Obtain the average value of two vertical angles.

5. Set the vertical circle vernier to zero and bring altitude bubble central. Take a reading on the staff held on the B.M. Take both face staff readings on B.M. to get average staff reading. Let it be S_1.
6. Shift the instrument to B. Set up and level it with reference to altitude bubble. Measure the vertical angle α_2 to Q with both face observations by repeating steps (3) and (4)
7. Repeat the step 5 and take the staffs reading S_2 on B.M. (if instruments axes at A & B are same level, then $S_1 = S_2$)

Depending upon the levels of instrument axes at P and R, the following cases are considered to calculate the R.L. of Q :

Case 1 : Instrument axes at A and B are at the same level.

Case 2 : Instrument axis at B is higher than that at A.

Case 3 : Instrument axis at A is higher than that at B.

Case 1 : When the instrument axes at *A* and *B* are at the same level

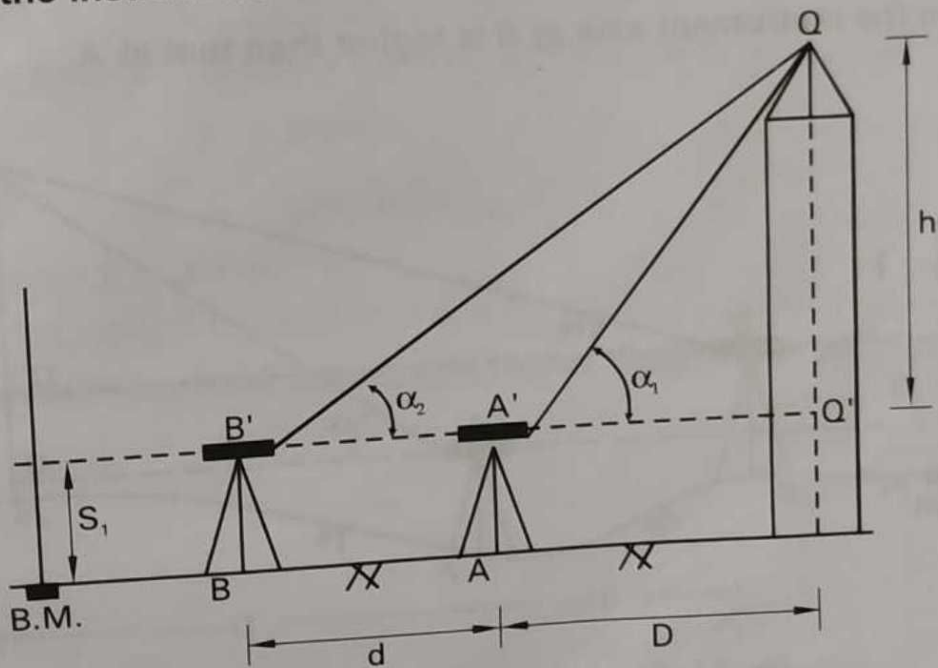

Fig. 2.2 : Instrument axes at the same level

Let, h be the height of Q above the horizontal line of sight $A'Q'$

α_1 = angle of elevation at A,

α_2 = angle of elevation at B,

S_1 = staff reading on B.M.

d = horizontal distance between A and B,

D = horizontal distance between A and Q.

From triangle $A'Q'Q$,

$$h = D \tan \alpha_1 \quad \text{---- (1)}$$

From triangle $B'Q'Q$,

$$h = (d + D)\ \tan\alpha_2 \text{ ---- } (2)$$

Equating equations (1) and (2)

$$D\ tan\alpha_1 = (d + D)\ \tan\alpha_2$$

or

$$\boxed{D = \frac{d\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2}}$$

Substitute the value of D in equation (1)

$$\boxed{h = \frac{d\tan\alpha_1\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2}}$$

Therefore, R.L of Q = R.L. of B.M. + $S_1 + h$

Case 2 : When the instrument axis at *B* is higher than that at *A*

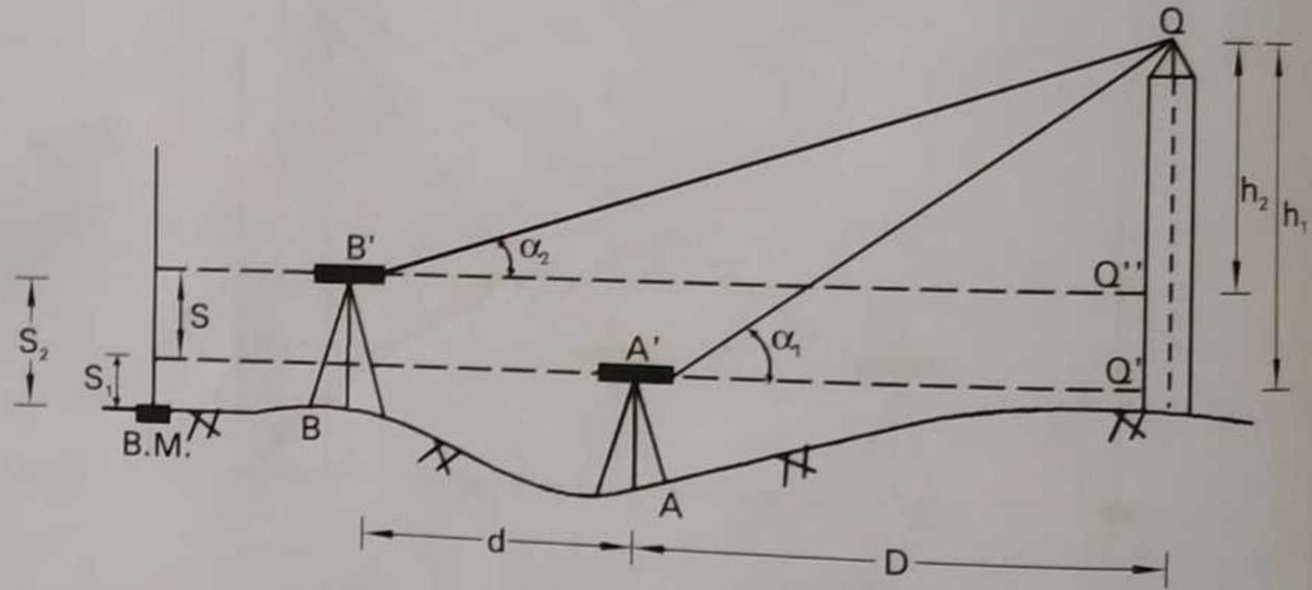

Fig. 2.3 : Instrument axis at B in higher than A

Refer fig. 2.3,

Let, S_1 = Staff reading on B.M. from the instrument station A

S_2 = Staff reading on B.M. from the instrument station B

Q' = Projection of Q on horizontal plane through A'

Q'' = Projection of Q on horizontal plane through B'

h_1 = height of Q above the horizontal plane through A'

h_2 = height of Q above the horizontal plane through B'

From triangle $A'QQ'$,

$$h_1 = D\ \tan\alpha_1 \text{ ---- } (1)$$

From triangle $B'QQ''$,

$$h_2 = (d + D)\ \tan\alpha_2 \text{ ---- } (2)$$

Subtracting equation (2) from (1)

$$h_1 - h_2 = D\ \tan\alpha_1 - (d + D)\tan\alpha_2$$

$$= D(\tan\alpha_1 - \tan\alpha_2) - d\ \tan\alpha_2$$

But, $h_1 - h_2$ = difference in levels of instrument axes

$$= S_2 - S_1 = S$$

therefore, $D\ (\tan\alpha_1 - \tan\alpha_1) - d\ \tan\alpha_2 = S$

or $D\ (\tan\alpha_1 - \tan\alpha_2) = S + d\ \tan\alpha_2$

$$\text{or}\ \boxed{D = \frac{d\tan\alpha_2 + S}{\tan\alpha_1 - \tan\alpha_2}}$$

Substitute the value of D in eqns. (1) and (2) to find h_1 and h_2.

$$h_1 = D\ \tan\alpha_1$$

$$h_2 = (d + D)\ \tan\alpha_2$$

Now, R.L. of Q = R.L of B.M. + $S_1 + h_1$

As a check, R.L. of Q = R.L of B.M. + $S_2 + h_2$

Case 3 : When the instrument axis at *A* is higher than that at *B*

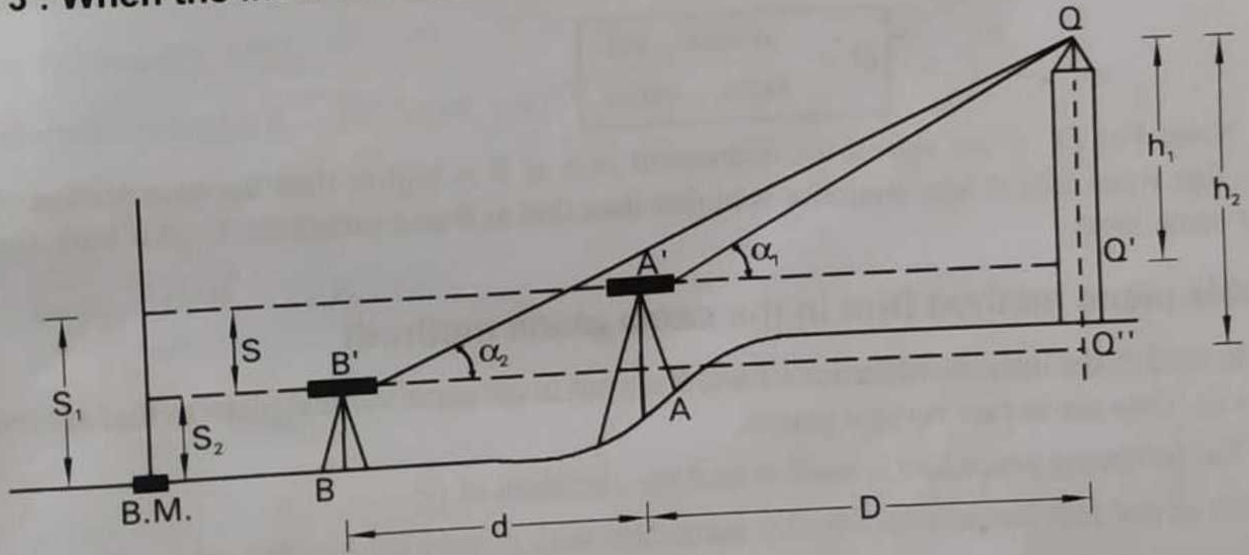

Fig. 2.4 : Instrument axis at A is higher than B

Refer fig.2.4,

From triangle $A'Q'Q$,

$h_1 = D\ \tan\alpha_1$ ---- (1)

From triangle $B'QQ'$,

$h_2 = (d + D) \tan\alpha_2$ ---- (2)

$h_2 - h_1 = (d + D) \tan\alpha_2 - D \tan\alpha_1$

$= d \tan\alpha_2 - D(\tan\alpha_1 - \tan\alpha_2)$

But, $h_2 - h_1$ = difference in levels of instrument axes

$= S_2 - S_1 = S$

Therefore, $S = d\ tan\alpha_2 - D(\tan\alpha_1 - \tan\alpha_2)$

or $D(\tan\alpha_1 - \tan\alpha_2) = d \tan\alpha_2 - S$

or

$$D = \frac{d\tan\alpha_2 - S}{\tan\alpha_1 - \tan\alpha_2}$$

Substitute the value of D in equations (1) and (2) to find h_1 and h_2.

$$h_1 = D\ tan\alpha_1$$

$$h_2 = (d + D)\ tan\alpha_2$$

R.L. of Q = R.L. of B.M. + $S_1 + h_1$

As a check, R.L. of Q = R.L. of B.M. + $S_2 + h_2$

Thus in general, the expressions for D, h_1 and h_2 can be written as

$$D = \frac{d\tan\alpha_2 \pm S}{\tan\alpha_1 - \tan\alpha_2}$$

Note: For D, (+)ve sign if the instrument axis at B is higher than the near station A, use (–)ve sign when axis at near station A is higher than that at B and substitute $S = 0$ if both axes are at the same level.

Double plane method (Not in the same plane method)

In fig.2.5, the instrument stations A and B are not in the same vertical plane as that of elevated object Q. They are in two vertical planes.

The following procedure is used to find the elevation of Q.

1. Set up the instrument ...

2.11 WORKED EXAMPLES

1. A transit is set up at 75 m away from a lightening conductor of a tall building. The angle of elevation to its top is 14°36'. The reading on a levelling staff held on a B.M of R.L. 1900.800 is 3.775m. Determine the R.L. of top of lightening conductor.

Solution:

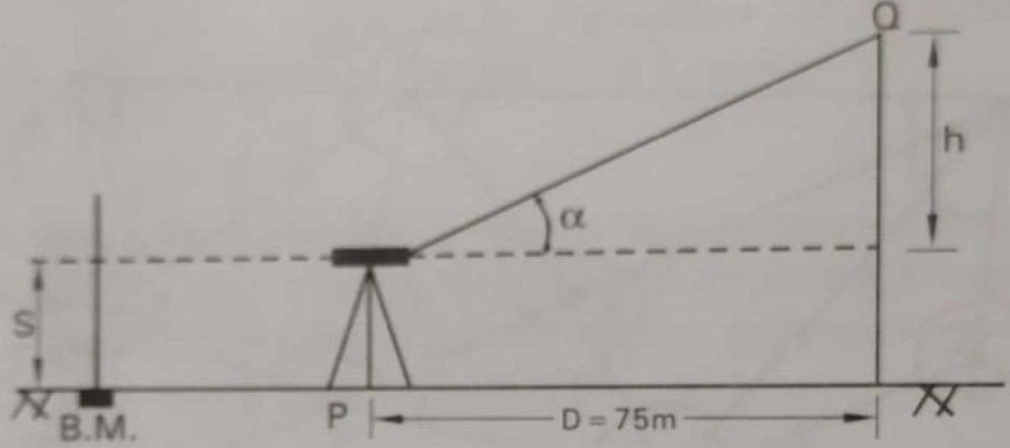

Fig. 2.12

Let , P = Instrument station.

Q = Top of lightening conductor.

Angle of elevation, $\alpha = 14^0 36'$, D = 75m, S = 3.775m

R.L. of instrument axis = R.L. of B.M + S =1900.800 + 3.775 = 1904.575m

$h = D \tan \alpha = 75 \times \tan 14^0 36' = 19.54m$

R.L. of top of lightening conductor = R.L. of instrument axis + h

= 1904.575 + 19.540 = 1924.115m

2. **Determine R.L. of top of flag pole from the following observations. The distance between stations & pole is 30m & R.L. of B.M. is 215.00 m.**

Station	Back sight	Vertical Angle to Top of Pole
A	2.05	30° 20'20"

Solution:

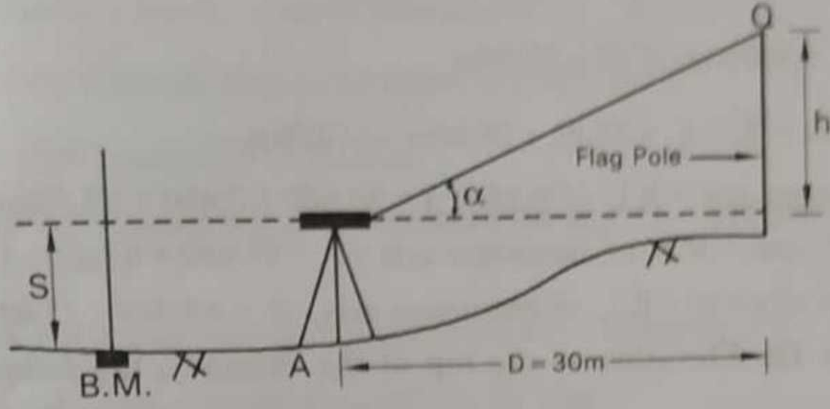

Fig. 2.13

A be the instrument station.

Let, Q be the top of flag pole.

Angle of elevation, $\alpha = 30^0 20'20''$, $D = 30\text{m}$, $S = 2.05\text{m}$

R.L. of instrument axis = R.L. of B.M. + S = 215.00 + 2.05 = 217.05m

$h = D\tan\alpha = 30 \times \tan 30^0 20'20'' = 17.56m$

R.L. of top of flag pole = R.L. of instrument axis + h = 217.05 + 17.56 = 234.61m

3. **A theodolite was set up at a distance of 500m from a tower and the angle of elevation to the top was 9°39'while the angle of depression to the foot of tower was 2°52'. The staff reading on a B.M. of R.L. 86.600 was 2.480m. What is the height of tower and R.L. of it's top and it's foot.**

Solution:

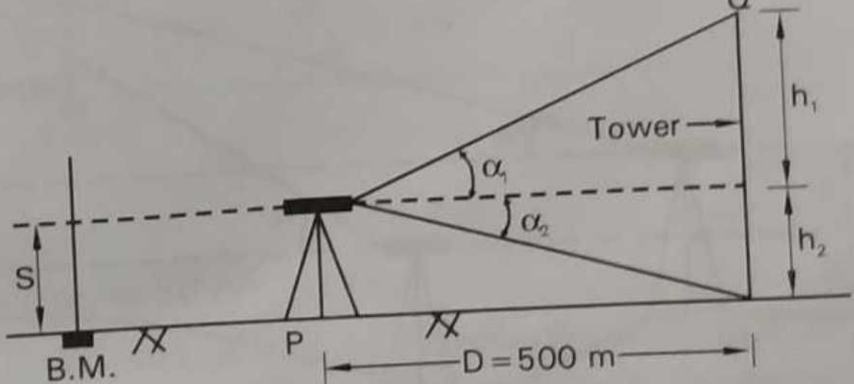

Fig. 2.14

Let, P be the instrument station and Q be the top of Tower

Angle of elevation, $\alpha_1 = 9^0 39'$

Angle of depression, $\alpha_2 = 2^0 52'$

$D = 500m$

$S = 2.480m$

$h_1 = D\tan\alpha_1 = 500\times\tan 9^0 39' = 85.02m$

$h_2 = D\tan\alpha_2 = 500\times\tan 2^0 52' = 25.04m$

Height of tower $= h_1 + h_2 = 85.02 + 25.04m = 110.06m$

R.L. of instrument axis = R.L. of B.M. + S = 86.600 + 2.480 = 89.080m

R.L. of top of tower = R.L. of instrument axis + h_1 = 89.080 + 85.02 = 174.100m

R.L. of bottom of tower = R.L. of instrument axis − h_2 = 89.080 − 25.040 = 64.040m

4. To determine the Elevation of the top of the chimney, the following observations were made:

Inst. Station	Reading on B.M.	Angle of elevation to Aerial Pole	Remarks
A	0.860	$18^0 36'$	R.L. of B.M.
B	1.220	$10^0 12'$	= 420.500m

Station *A* & *B* and the top of the chimney are in the same vertical plane. Find the Elevations of the top of the chimney, if the distance between *A* & *B* is 50.00m.

Solution:

$S_1 = 0.860m, S_2 = 1.220m, \alpha_1 = 18^0 36', \alpha_2 = 10^0 12', d = 50m.$

Instrument axis at B is higher than A ($S_2 > S_1$).

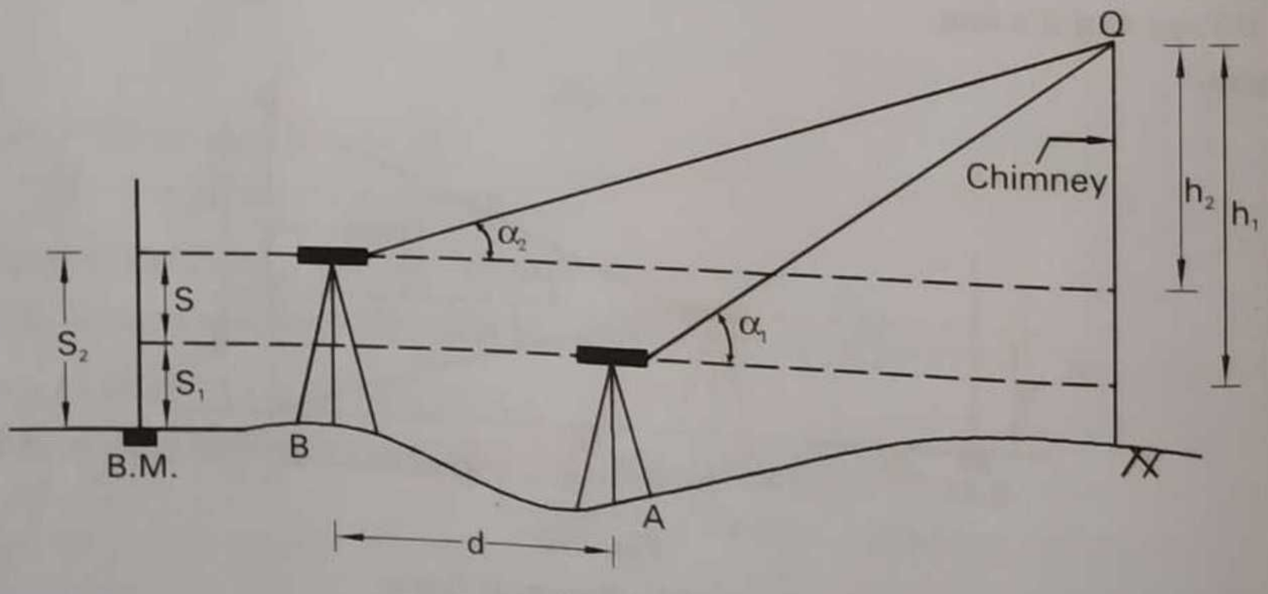

Fig. 2.15

R.L. of instrument axis at A = R.L. of B.M. + S_1 = 420.500 + 0.860 = 421.360m

R.L. of instrument axis at B = R.L. of B.M. + S_2 = 420.500 + 1.220 = 421.720m

Difference in level of instrument axis = $S = S_2 - S_1$ = 1.220 – 0.860 = 0.36m

$$\text{Distance, } D = \frac{d\tan\alpha_2 + S}{\tan\alpha_1 - \tan\alpha_2} = \frac{50\times\tan 10^0 12' + 0.360}{\tan 18^0 36' - \tan 10^0 12'} = \frac{9.356}{0.157} = 59.60m$$

Height of top of the chimney above instrument axis through A

$h_1 = D\tan\alpha_1 = 59.60\times\tan 18^0 36' = 20.06m$

R.L. of the top of the chimney = R.L. of instrument axis at $A + h_1$ = 421.360 + 20.060 = 441.420m

Check:

Height of top of the Aerial pole above instrument axis through B

$h_2 = (d + D)\tan\alpha_2 = (50 + 59.60)\times\tan 10^0 12' = 19.72m$

R.L. of the top of the chimney = R.L. of instrument axis at $B + h_2$ = 421.72 + 19.72 = 441.440m

5. An instrument was set up at A and the angle of elevation of the top of an electric pole BC was 24° 36'. The horizontal distance between A and B, the foot of the pole was 600 m. Determine the reduced level of the top of the pole, if the staff reading held on B.M. (R.L. 100.00 m) was 2.532 m, with the telescope in horizontal plane.

Solution:

$\alpha = 24^0 36'$, $D = 600\text{m}$, $S = 2.532\text{m}$.

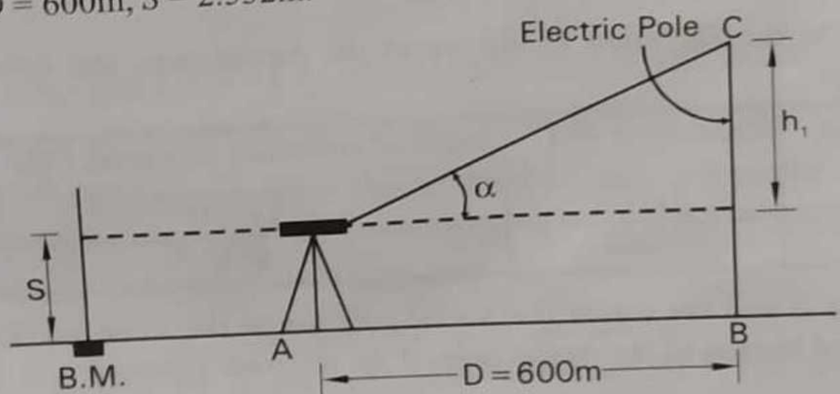

Fig. 2.16

$h_1 = D\tan\alpha = 600\times\tan 24^0 36' = 274.702m$

Therefore, R.L. of C = R.L. of B.M. + $S + h_1$ = 100.00 + 2.532 + 274.702 = 377.234m

6. A theodolite was set up in between two towers X and Y. The distance of the theodolite station from X is 60 m and from Y is 120 m. Observations were taken from theodolite to the top of towers X and Y and were recorded as 33° 26' 20" and 30° 50' 40" respectively, the telescope focussed upwards for both the cases. The R.L. of the trunion axis of the

theodolite was 139.675 m above the M.S.L. Calculate the R.L. of the top of the tower X and that of Y.

Solution:

$D_1 = 60\text{m}, D_2 = 120\text{m}, \alpha_1 = 33^0 26'20'', \alpha_2 = 30^0 50'40''$

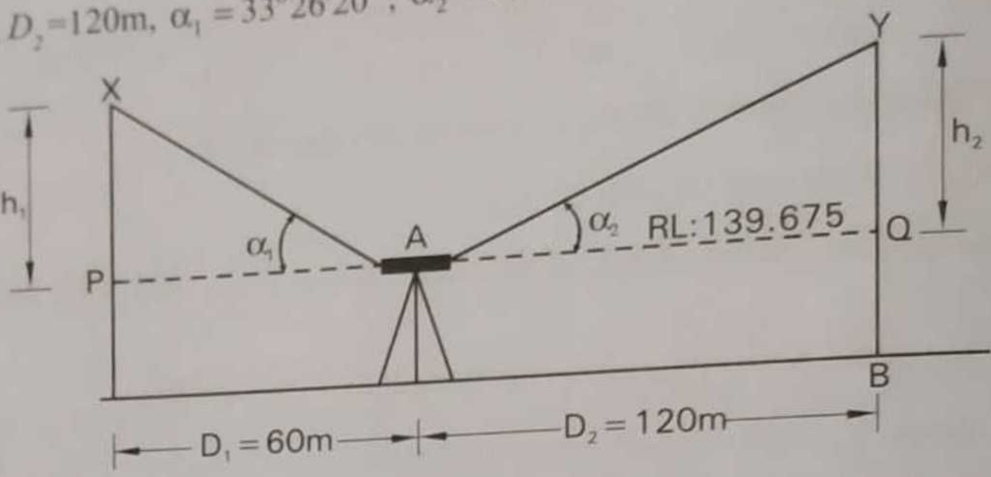

Fig. 2.17

Let, PQ be the horizontal line through the trunnion axis of the theodolite A.

From triangle XAP, $h_1 = D_1 \tan \alpha_1 = 60 \times \tan 33^0 26'20'' = 39.621m$

From triangle YAQ, $h_2 = D_2 \tan \alpha_2 = 120 \times \tan 30^0 50'40'' = 71.660m$

R.L. of top of tower X = R.L. of trunnion axis + h_1 = 139.675 + 39.621 = 179.296m

R.L. of top of tower Y = R.L. of trunnion axis + h_2 = 139.675 + 71.660 = 211.335m

7. To determine the Elevation of the top of the Aerial pole, the following observations were made:

Inst. Station	Reading on BM	Angle of elevation to Aerial Pole	Remarks
A	1.377	$11^0 53'$	R.L. of B.M. = 30.150m
B	1.263	$8^0 5'$	

Station A & B and the top of the Aerial pole are in the same vertical plane. Find the Elevations of the top of the Aerial pole, if the distance between A & B is 30.00m.

Solution:

$S_1 = 1.377\text{m}, S_2 = 1.263\text{m}, \alpha_1 = 11^0 53', \alpha_2 = 8^0 5', d = 30\text{m}.$

Instrument axis at A is higher than B ($S_1 > S_2$).

R.L. of instrument axis at A = R.L. of B.M. + S_1 = 30.150 + 1.377 = 31.527m

R.L. of instrument axis at B = R.L. of B.M. + S_2 = 30.150 + 1.263 = 31.413m

Difference in level of instrument axis = $S = S_1 - S_2$ = 1.377 − 1.263 = 0.114m

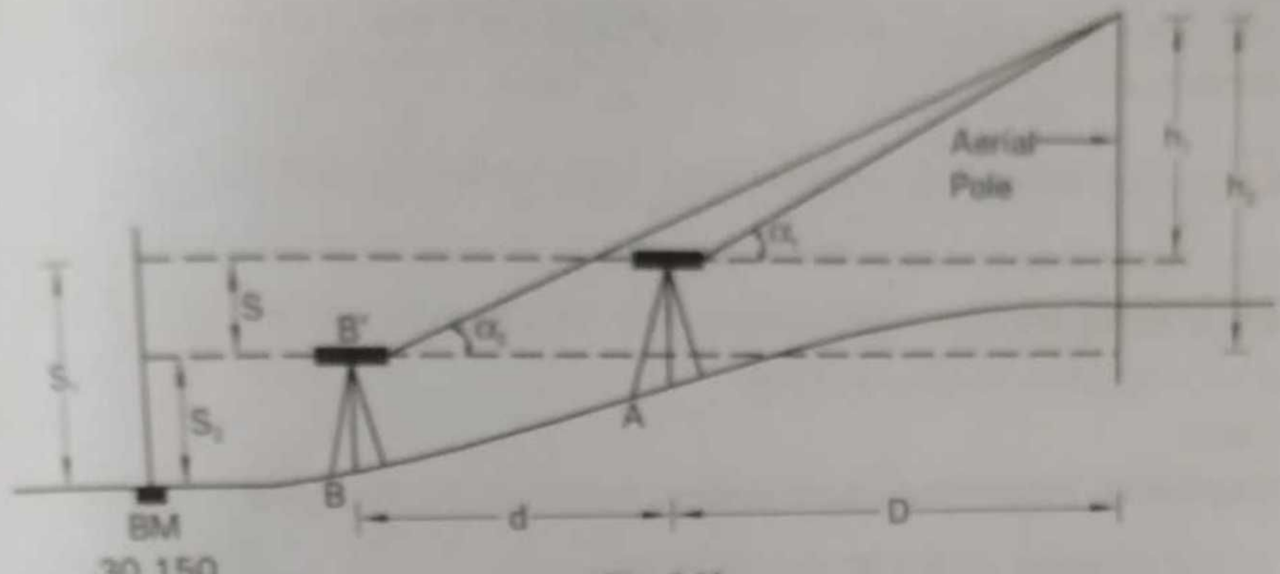

Fig. 2.18

$$\text{Distance, } D = \frac{d\tan\alpha_2 - S}{\tan\alpha_1 - \tan\alpha_2} = \frac{30\times\tan 8^\circ 5' - 0.114}{\tan 11^\circ 53' - \tan 8^\circ 5'} = \frac{4.26 - 0.114}{0.0684} = 60.62m$$

Height of top of the Aerial pole above instrument axis through A

$h_1 = D\tan\alpha_1 = 60.62\times\tan 11^\circ 53' = 12.757m$

R.L. of the top of the Aerial pole = R.L. of instrument axis at $A + h_1 = 31.527 + 12.757 = 44.284$m

Check:

Height of top of the Aerial pole above instrument axis through B

$h_2 = (d + D)\tan\alpha_2 = (30 + 60.62)\times\tan 8^\circ 5' = 12.870m$

R.L. of the top of the Aerial pole = R.L. of instrument axis at $B + h_2 = 31.413 + 12.870 = 44.284$m

8. Find the elevation of the top of a lightning conductor from the following data:

Inst. Station	Reading on BM	Angle of Elevation	Remarks
A	0.648	$22^\circ 0'$	R.L. of B.M.= 150.650 m
B	0.984	$14^\circ 35'$	Distance, AB = 21.00 m

Station A & B and the top of the lightening conductor are in the same vertical plane.

Solution:

$S_1 = 0.648$m, $S_2 = 0.984$m, $\alpha_1 = 22^\circ 0'$, $\alpha_2 = 14^\circ 35'$, $d = 21$m.

Instrument axis at B is higher than A ($S_2 > S_1$).

R.L. of instrument axis at A = R.L. of B.M. + S_1 = 150.650 + 0.648 = 151.298m

R.L. of instrument axis at B = R.L. of B.M. + S_2 = 150.650 + 0.984 = 151.634m

Difference in level of instrument axis = $S = S_2 - S_1 = 0.984 - 0.648 = 0.336$m

Distance, $D = \frac{d \tan\alpha_2 + S}{\tan\alpha_1 - \tan\alpha_2} = \frac{21 \times \tan 14^0 35' + 0.336}{\tan 22^0 0' - \tan 14^0 35'} = \frac{5.46 + .336}{0.144} = 40.25m$

Height of top of lightening conductor above instrument axis through A

$h_1 = D \tan\alpha_1 = 40.25 \times \tan 22^0 0' = 16.26m$

R.L. of the top of a lightening conductor = R.L. of instrument axis at

$A + h_1 = 151.298 + 16.26 = 167.558$m

Check:

Height of top of lightening conductor above instrument axis through B

$h_2 = (d + D) \tan\alpha_2 = (21 + 40.25) \times \tan 14^0 35' = 15.93m$

R.L. of the top of a lightening conductor = R.L. of instrument axis at

$B + h_2 = 151.634 + 15.93 = 167.564$m

9. Determine the top of a tower from the following observations and also the distance from *B* :

Inst. Station	Reading on B.M.	Vertical angle of Tower	R.L. of BM.
B A	3.525 2.000	$16^0 30'$ $10^0 30'$	325.00 m

Distance between *B* and *A* = 50m.

Solution:

$S_1 = 3.525$m, $S_2 = 2.000$m, $\alpha_1 = 16^0 30'$, $\alpha_2 = 10^0 30'$, $d = 50$m.

Instrument axis at B is higher than A ($S_1 > S_2$).

R.L. of instrument axis at B = R.L. of B.M. + S_1 = 325.00 + 3.525 = 328.525m

R.L. of instrument axis at A = R.L. of B.M. + S_2 = 325.00 + 2.000 = 327.000m

Difference in level of instrument axis = $S = S_1 - S_2 = 3.525 - 2.000 = 1.525$m

Distance, $D = \frac{d \tan\alpha_2 - S}{\tan\alpha_1 - \tan\alpha_2} = \frac{50 \times \tan 10^0 30' - 1.525}{\tan 16^0 30' - \tan 10^0 30'} = \frac{7.741}{0.111} = 69.74m$

Height of top of the tower above instrument axis through B

$h_1 = D \tan\alpha_1 = 69.74 \times \tan 16^0 30' = 20.66m$

R.L. of the top of the tower = R.L. of instrument axis at $B + h_1 = 328.525 + 20.66 = 349.185$m

Check:

Height of top of the tower above instrument axis through A

$h_2 = (d + D) \tan\alpha_2 = (50 + 69.74) \times \tan 10^0 30' = 22.19m$

R.L. of the top of tower = R.L. of instrument axis at $A + h_2 = 327.000 + 22.19 = 349.19$m

CONCEPT REVIEW QUESTIONS

I Descriptive type questions

1. What is Trigonometrical levelling and what are its practical applications?
2. Explain the basic principles of trigonometric levelling.
3. Differentiate between Trignometrical levelling and Ordinary levelling.
4. What is meant by trigonometric levelling? Where is it employed?
5. What are the practical applications of Trignometrical levelling.
6. Explain the procedure to find the RL of an elevated object whose base is inaccessible when the instrument axes are at the same level.
7. Explain the procedure to find the RL of an elevated object whose base is accessible by single plane method.
8. How do you find the Elevation of an object whose base is inaccessible when the instrument axes are at the same level ?
9. Define triangulation. Explain its purpose.
10. Explain classification of triangulation.

QUESTIONS FOR SELF EXAMINATION

II Fill up the blanks type questions

1. The simplest figure in a triangulation network is ____________
2. The most precise horizontal control is obtained by____________
3. The first order triangulation is of the____________ order.
4. The best shape of well-conditioned triangle is isosceles with base angle equal to ____________

III True or false type questions

1. In trigonometric leveling, If the distances are small, the effect of curvature and refraction is neglected.
2. In trigonometric survey, the vertical angles are usually measured with a dumpy level and the horizontal distances are measured with a tape or chain or calculated indirectly.
3. If the two instrument stations so chosen lie in the same vertical plane passing through the elevated object, it is known as "single plane method".
4. The basic figures used in triangulation networks are the triangle, braced or geodetic quadrilateral, and the polygon.
5. The first order triangulation consists of a number of points fixed within the framework of primary triangulation.

IV Trignometrical levelling Quiz

1. The process of determining the difference of elevations of stations from vertical angles and known distances is known as
   (a)Trignometric levelling (b)Geodetic surveying (c)Field astronomy (d)Topographic surveying
2. Triangulation surveys are carried out for locating
   (a)Control points for surveys of large areas
   (b)Control points for photogrammetric surveys
   (c)Engineering works, i.e., terminal points of long tunnels, bridge, abutments, etc.
   (d)All of the above
3. Trigonometrical levelling the cases encountered are :
   (a)Base of the object accessible (b) Base of the object inaccessible (c) Both (a) & (b)
   (d)none of these
4. If it is possible to measure the horizontal distance from the instrumentation to the object__________ method preferred.
   (a)Direct method (b) Single plane method (c)Double plane method (d)Any one of these

5. In Single plane method, if α is the angle of elevation and D is the distance between the object and instrumentation, then Height of the object given by

(a) $\frac{\tan\alpha}{D}$ (b) $\frac{D}{\tan\alpha}$ (c) $D\sin\alpha$ (d) $D\tan\alpha$

6. The permissible average and maximum triangular closure in seconds in a second order triangulation is

(a) 1" and 3" (b) 2" and 3" (c) 3" and 4" (d) 3" and 5"

7. The best shape of a triangle in triangulation is

(a) equilateral (b) isosceles with base angle 56° 14' (c) isosceles with base angle 65" 14' (d) isosceles with base angle 30°

8. The limiting strength of figure for first order and third order triangulations are

(a) 25 and 30 (b) 25 and 50 (c) 30 and 25 (d) 50 and 25

9. The number of stations for a triangulation framework can be reduced, yet maintaining the required accuracy, by using a chain of

(a) triangles (b) pentagons (c) parallelograms (d) polygons

V Problems for practice

1. An instrument was set up at *A* and the angle of elevation of the top of a tower BC was 26°15'. The horizontal distance *AB*, *B* being the foot of the tower, was 715 m. Determine the R.L. of the top of the tower if the staff reading held on a station *P* of R.L. 100.00 m was 2.455 with the telescope horizontal.
2. A theodolite was set up on the top of a hill 3000m away from the base of a tower, down hill, the angle of depression being 18°24'. The height of the theodolite as read on a staff held on B.M. of R.L. 2175.250 was 2.665m. Calculate the R.L. of base of the tower.
3. The theodolite was set up at a distance 200m away from a tower. The angle of elevation to the top of the parapet was 8°18', while the angle of depression to the foot of the tower was 2°24'. The staff reading on the B.M. of R.L. 248.360 with the telescope horizontal was 1.290m. Find the height of the tower and R.L. of top of the parapet.
4. To determine the Elevation of the top of the chimney, the following observations were made:

Inst. Station	Reading on B.M.	Angle of elevation to Aerial Pole	Remarks
P	2.625	19°48'	R.L. of B.M. = 100.000m
Q	1.510	14°25'	

Station *P* & *Q* and the top of the chimney are in the same vertical plane. Find the Elevations of the top of the chimney, if the distance between *P* & *Q* is 50.00m.

5. Determine the R.L. of top of the tower from the following data:

Inst. Station	Reading on B.M.	Angle of elevation to Aerial Pole	Remarks
A	3.625	$+16^0 42'$	R.L. of B.M. = 1728.785m
B	2.005	$+11^0 12'$	

Station A & B are in line with top of the tower. Distance AB = 30m.

6. To determine the elevation of a point A, on the top of a hill a flag staff AB of 3 m height, was erected and the observations from two stations M and N, 50 m apart, were made as given below:

Horizontal angle between B and N at $M = 65^0 25'$

Horizontal angle between B and M at $N = 72^0 30'$

Angle of elevation of B from $M = 12^0 24' 29''$

Angle of elevation of B from $N = 12^0 34' 32''$

Staff reading on B.M. when instrument at M = 1.785 m

Staff reading on the same B.M. when instrument at N = 2.305 m

If the R.L. of the B.M. is 200.00 m. what is the R.L. of A?

VII Activity

Determine reduced levels and heights of objects like chimneys, towers or any other tall structures and compared results by both single plane and Double plane methods.

UNIT 2- TRIGONOMETRIC LEVELLING

Cognitive level –Remember

1. Explain the basic principles of Trignometrical Levelling.
2. Differentiate between Trignometrical levelling and Ordinary levelling.
3. What is trigonometrical levelling? Where it is employed?

Cognitive level –Understand

3. What are the practical applications of Trignometrical levelling.
4. Explain the procedure to find the RL of an elevated object whose base is inaccessible when the instrument axes are at the same level.
5. Explain the procedure to find the RL of an elevated object whose base is accessible by single plane method.
6. Explain briefly the method of determining the height of an object by double plane method.

Cognitive level –Application

7. An instrument was set up at P and the angle of elevation to a vane 4m above the foot of the staff held at Q was $9^0 30'$. The horizontal distance between P & Q is 2000m. Determine the RL of the Staff station Q given that RL of the instrument axis is 2650.38m by single plane method.
8. Find the RL of the church Spire C from the following observations taken from two stations A and B, 50m apart,

   Angle BAC = 60^0

   Angle ABC = 50^0

   Angle of elevation from A to top of Spire = 30^0

   Angle of elevation from B to top of spire = 29^0

Staff reading from A on BM = 2.5m

Staff reading from B on BM = 0.5m

RL of BM = 20m.

9. A transit theodolite was set up at a distance of 200m from a chimney and angle of elevation to its top was $10^0 48"$. The staff reading on a BM of RL 70.250m with the telescope horizontal was 0.977. Find the RL of top of Chimney.

UNIT 3- TACHEOMETRY

Cognitive level –Remember

1. Explain the Principle of Tacheometry.
2. What are the purpose of Tacheometry.
3. What is Anallatic lense.
4. What is a Stadia Rod.